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Quantum Computation with Aharonov-Bohm Qubits

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Abstract

We analyze the possibility of employing a mesoscopic/nanoscopic ring of a normal metal in a double-degenerate persistent-current state **with a third auxiliary level and** in the presence of the Aharonov-Bohm flux equal to the half of the normal flux quantum, hc/e , as a qubit. The auxiliary level can be effectively used for all fundamental quantum logic gate (qu-gate) operations which includes the initialization, phase rotation, bit-flip, and the Hadamard transformation, as well as the double-qubit controlled operations (conditional bit flip). We suggest a tentative realization of the mechanism proposed as either the mesoscopic structure of three quantum dots coherently coupled by resonant tunneling in crossed magnetic and electric fields, or as a nanoscopic structure of triple anionic vacancy (similar to F_3 centers in alkali halides) with one trapped electron in one spin projection state.

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I. INTRODUCTION

Quantum computation [1-3] is a promising tool for solving intractable mathematical problems, those in which the number of computational steps (if solved with a classical computer) increases exponentially with the number of computational units (N), e.g., number of spins in the Heisenberg ferromagnet, number of electrons and lattice sites in the Hubbard model of solid, number of binary digits in a large integer to be factorized, etc. If these units (spins, atoms, digits) are represented as “quantum bits” [4] and processed by unitary transformations acted upon by the logical quantum gates [5], at least some of these problems can be solved in a polynomial time in N (e.g., the Shor’s algorithm [6] for factorizing large integers). Basically, the fundamental gates are unitary time evolutions for given Hamiltonians executed on qubits or on pairs of qubits and for certain time intervals. Fundamental gates are known to be the unitary operations such as the bit-flip, phase-flip, Hadamard and the controlled-NOT (CNOT) operations. Workers in the field at earlier times considered qubit realizations as quantum optical or atomic systems, and shifted at more recent times to other methods employing mesoscopic condensed matter structures (quantum dots [7], superconducting Cooper-pair boxes [8-11], Josephson junctions [12-14]) as qubits). In Ref.[15], the necessary conditions for quantum computation have been specified, not all of which have already achieved perfect realization (the problems with the solid-state qubits are documented in Refs.[16,17]). This leaves space for more suggestions of the instrumental realization of qubits, especially those that use the solid state technology. We investigate in this paper one of such possibilities. The particular advantage of the suggested three state mesoscopic persistent current system is the fact that no physical coupling or switch is required in the manifestation in any of the fundamental quantum gates. (Similar ideas have been suggested earlier [18], however without any clear instrumental realization.) In addition, the auxiliary level can also be used to coherently couple the operational qubit states to the environment including the other qu-gates as well as the input-output devices, and hence it is expected to help also in the manifestation of possible error correction mechanisms. The proposed

structure is naturally realized with the quantum states of the ring of metallic islands (or atomic sites) connected by resonant tunnelling in the presence of the Aharonov-Bohm flux threading the ring, a persistent-current (PC) loop [19], and placed in an external electric field perpendicular to the magnetic flux to perform the qu-gate manipulation in the invariant subspace of two degenerate states. We focus on the quantum mechanical aspects of qu-bit and qu-gate operations with the PC loops leaving for future the discussion on their practical implementation.

II. PERSISTENT-CURRENT QUBIT

In the mesoscopic ring of a normal metal of size L smaller than the phase-decoherence length of the electrons, the charge current is produced under the influence of Aharonov-Bohm (AB) flux [20]. Physically, the shifted energy minimum in the presence of AB flux is counterbalanced by a net charge flow producing a persistent current in the absence of resistive effects. This phenomenon was predicted in [21], rediscovered in [22], and probed in an experiment in [23-25]. The magnitude of persistent current in a clean metallic ring is typically given by $J_c \sim ev_F/L$ where v_F is the electron Fermi velocity. In a nanoscopic (atomically small) ring with one electron, the magnitude of the maximal persistent current is $J_c \sim e|t_0|/\hbar N^2$ where N is the number of sites in a ring and t_0 is the electron hopping amplitude between the sites. The PC is created individually by single electrons hence the fundamental flux quantum $\Phi_0 = hc/e$ is twice larger than the Abrikosov or Josephson flux quantum $\Phi_1 = hc/2e$. This very fact may permit new effects to arise when a single Josephson vortex or Abrikosov fluxon is used to manipulate the normal flux in a PC ring. Particularly, it is known that at $\Phi = \Phi_1$ the PC ring has a degenerate ground state configuration which produces a frustrated (entangled) quantum state. Similar to the recently proposed superconducting qubits manipulating macroscopic coherence, the qubit in the persistent normal current ring is defined in this doubly degenerate ground state configuration with the exception of the

auxiliary level in our case. The Hamiltonian of the system is

$$H = -T \sum_{n=1}^N (a_n^+ a_{n+1} e^{i\alpha} + a_{n+1}^+ a_n e^{-i\alpha}) \quad (1)$$

where a_n^+ is a fermionic operator creating (and a_n , annihilating) electron at site \mathbf{R}_n in a ring, $a_{N+1} = a_1$, and α is the Aharonov-Bohm phase related to magnetic flux threading the ring by $\alpha = 2\pi\Phi/N\Phi_0$. The Hamiltonian (1) is diagonalized by the angular momentum (i.e., $m = 0, 1, \dots$) eigenstates $A_m^+|0\rangle$ where

$$A_m^+ = \frac{1}{\sqrt{N}} \sum_{n=1}^N e^{2\pi i m n / N} a_n^+ \quad (2)$$

with the site energies $\varepsilon_m = -2T \cos \frac{2\pi}{N}(m + \frac{\Phi}{\Phi_0})$ plotted against the normalized flux Φ/Φ_0 in Fig.1. In our case here, the number of sites $N = 3$.

Since two ground states are degenerate at $\Phi_0/2$, they can be used as the components of qubit while the third one is coupling the qubit to a qu-gate, to be discussed in the next two sections. The practical realization of the qubit with desired architecture is sketched in Fig.1. One possibility may be a three-sectional normal-metal ring intersected by tunnelling barriers (or consisting of overlapping metallic films separated by thin oxide layers). Creating strong magnetic field to operate the qubit at the half quantum flux is suggested with the help of superconducting fluxon trapped in a hole inside the superconducting film, with lines of magnetic field further focused by a mesoscopic ferromagnetic cylinder near the ring.

The isolated qubit structure can in principle be realized as a three-site defect in an insulating crystal, similar to negative-ion triple vacancy (known as F_3 -center) in the alkali halide crystals (e.g., see [26]). The gate manipulation in such a structure is provided by applying the electric field perpendicular to the Aharonov-Bohm flux which will allow performing operations in an invariant subspace of two degenerate states (clockwise and counterclockwise persistent current directions).

III. QUBIT OPERATIONS IN AN INVARIANT SUBSPACE OF ONE RING

In the terms of operators A_m^+ , the Hamiltonian (1) is transformed into the diagonal form (we scale all energies in units of T)

$$H_0 = \sum_m \varepsilon_m A_m^+ A_m = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (3)$$

The interaction with an electrostatic potential $V_n = V_0 \cos(2\pi n/3)$ at site n , introduced with potential electrodes inclosing the ring (Fig.2), is presented with the interaction term

$$H_1(V_0) = \frac{V_0}{2} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}. \quad (4)$$

We also introduce, for further use, the interaction with a static site potential V_S (the Hamiltonian H_2), and the Hamiltonian H_3 representing the effect of the shifted magnetic flux away from $\Phi_0/2$ on the eigenenergy levels $\Delta f = (\Phi - \Phi_0/2)/\Phi_0$

$$H_2 = V_S \text{diag}(1, 1, 1) \text{ and } H_3 = \text{diag}(\Delta\varepsilon_1, \Delta\varepsilon_2, \Delta\varepsilon_3) \quad (5)$$

(remember that V_0, V_S are in units of T). It is shown below that the first two Hamiltonians H_0 and H_1 suffice in the realization of the fundamental gates except the phase shift. The H_2 and H_3 perform relative phase shift between the qubit states. The time dependence of the amplitudes $C_n(t)$ in the angular momentum basis are found as

$$C_n(t) = \sum_m [\exp(-iHt)]_{mn} C_m(0) \quad (6)$$

where, in general, $H = (H_0 + H_1 + H_2 + H_3)$. At this moment, we consider $V_S = 0$ and $\Phi = \Phi_0/2$ hence H_2 and H_3 do not exist. When the interaction H_1 is turned on between $t = 0$ and t and zero otherwise, the amplitudes are found by

$$C_n(t) = \sum_{m,k} S_{kn}^{-1}(V_0) e^{-iE_k t} S_{mk}(V_0) C_m(0) \quad (7)$$

where $E_k(V_0)$ are the eigenenergies of Hamiltonian $H_0 + H_1(V_0)$ and $S_{nm}(V_0)$ are the unitary matrices transforming from the noninteracting eigenbasis (the ones corresponding to H_0) to the eigenbasis of the full Hamiltonian $H_0 + H_1$. Inspection of Eq.(7) shows that, at fixed values of V_0 and the evolution time t , the population of the auxiliary state (in the noninteracting basis) vanishes if it was initially set to zero. This happens when the three energies $\varepsilon_n(V_0)$, ($n = 1, 2, 3$) are commensurate, so that exponential factors in Eq.(7) destructively interfere with each other at fixed instants in time.

The eigenenergies $E_k(V_0)$ are plotted in Fig.3. The amplitudes vanish when the commensurability condition

$$E_3 - E_1 = K(E_2 - E_3) \quad (8)$$

is satisfied for integer K . The corresponding values of the potential that does Eq.(8) are

$$V_0(K) = -\frac{2}{3K}[K^2 + K + 1 + (K - 1)\sqrt{K^2 + 4K + 1}]. \quad (9)$$

In particular we mention that for $K = 1$ one has $V_0^{(1)} = -2$; and at $K = 3$ one has $V_0^{(3)} = -\frac{2}{9}(13 + 2\sqrt{22}) = -4.9735$. As shown below these two cases give the bit-flip and Hadamard transformations. The $K = 1$ case can be explicitly proved by checking the identity

$$\exp\left\{-it \begin{pmatrix} -1 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & -1 \end{pmatrix}\right\} = \frac{1}{2} \begin{pmatrix} 1 + c + s & s & -1 + c + s \\ s & 2(c - s) & s \\ -1 + c + s & s & 1 + c + s \end{pmatrix} \quad (10)$$

where $c = \cos(t\sqrt{6})$, $s = i\sqrt{\frac{2}{3}}\sin(t\sqrt{6})$. At $s = 0$ (i.e. $c = 1$), the transformation matrix of Eq.(10) block-diagonalizes in a subspace of states 1,3 (i.e. $|0\rangle, |1\rangle$) and the upper state 2 (i.e. $|c\rangle$), after reshuffling state numbering from (1,2,3) to (1,3,2).

In Fig.4 the populations of the states $p_n(t) = |C_n(t)|^2$ are plotted for these two cases $K = 1$ and $K = 3$. At times t_1 for $K = 1$ and t_3 for $K = 3$, the population of the auxiliary state vanishes assuming that it was zero at $t = 0$. These special times are (in units of \hbar/T)

$$t_1 = \frac{\pi}{\sqrt{6}} = 1.2825, \quad t_3 = \frac{\pi}{2[\varepsilon_2(V) - \varepsilon_3(V)]_{V=V_0(3)}} = 0.7043 \quad (11)$$

where

$$\varepsilon_{1,3}(V) = \frac{1+V/2}{2} \mp \frac{3}{2}\sqrt{1-V/2+V^2/4}, \quad \varepsilon_2(V) = -1 - V/2 \quad (12)$$

for $V \leq 0$. We notice that the configuration $(t_1, K=1)$ performs the bit-flip $|0\rangle \leftrightarrow |1\rangle$ whereas $(t_3, K=3)$ creates equally populated Hadamard-like superpositions of $|0\rangle$ and $|1\rangle$. These operations are presented in the qubit subspace by the matrices (overall phases are not shown)

$$G_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } G_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}. \quad (13)$$

The G_1 gate manifests the bit-flip whereas G_3 is different from the standard Hadamard by a relative $\pi/2$ phase. The relative phase in G_3 can be changed by an additional procedure using the diagonal Hamiltonians H_2 and H_3 defined above by introducing additional relative energy difference between the qubit states without changing the couplings and the eigenstates (note that H_0 , H_2 and H_3 are simultaneously diagonal). If the operational space is a double-qubit space the overall phase of the qubit states becomes important. For instance, the overall qubit phase can be corrected with the unitary matrix $\exp(-i(H_0 + H_2)t)$ at a fixed and determinable configuration $(t^*, V_0^{(*)})$ whereas the relative phase between the qubit states can be changed using the phase rotation matrix

$$G_2(\phi) = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} \quad (14)$$

in the form of an Euler-type transformation $G_2(-\pi/4) G_3 G_2(-\pi/4)$. The fixed phase value $-\pi/4$ can be obtained by turning off H_1 and H_2 and turning on H_3 [i.e. $H = (H_0, 0, 0, H_3)$] for the required time. Since both Hamiltonians are diagonal, the qubit subspace is invariant under this transformations for all evolution times.

The gate operations described in this section can be regarded as the (quenched) Rabi oscillations affected by the nondiagonal matrix elements generated by H_1 . The transitions between the degenerate states are achieved through virtual transitions to an auxiliary eigenstate $|c\rangle$ with a sufficiently higher energy level (Fig.5). Switching off the interaction, when the

auxiliary state becomes depopulated, brings the final configuration back to the qubit subspace.

IV. QUANTUM COMPUTING PROCEDURES WITH ONE AND TWO PERSISTENT-CURRENT RINGS

The standard procedures of quantum computation are the initialization (input), the logic gate transformations in one ring, the controlled bit flips on the qubit pairs (the *CNOT*), and the reading of the output to a classical device. We discussed the bit-flip, Hadamard as well as the phase shift above. Here, we discuss the initialization, the two-qubit CNOT gate and the read-out operations.

(a)*Initialization.* Adiabatically shift the magnetic flux in each ring from half flux quantum and allow the system to relax to the nondegenerate lowest energy state $|0\rangle$ by spontaneous emission. By applying G_3 , we receive a state of equally superposed degenerate levels which is conventionally the initial state in some quantum computing algorithms, in particular Shor's algorithm for factorizing large integers [1].

(b)*CNOT.* This gate performs a conditional bit-flip in the qubit No.2 when the qubit No.1 is in a fixed state. To make such a transformation a non-demolition measurement of the state of the first qubit is needed. We use the direction of the current in the first qubit to control the bit flip in the second qubit. The suggested scheme is sketched in Fig.6. The flux in the qubit No.1 (which includes the externally applied flux and the flux created by a persistent current) is extracted from the former by a $-\Phi_0/2$ compensating coil, and further supplied to the Hall bar with a (large) current passing through it. The Hall voltage generated in the bar, after extraction of the voltage corresponding to fixed value related to the counterclockwise current direction and applied to the V electrodes of qubit No.2 produces,

if the voltage proves nonzero, the flip of the second qubit. The procedure may in principle be executed in a totally reversible way if the Quantum Hall Effect regime is manifest [29]. In more detail, assume that the current generated by a gubit loop, (J_Q), is $J_Q = J_0$ when the qubit is in the clockwise direction of rotation (a state $|0\rangle$), and value of the current $J_Q = -J_0$ when the direction of rotation is counterclockwise (i.e., qubit is in a state $|1\rangle$). Supply a current thus received from the first qubit, with a fixed value J_0 added, $J_Q + J_0$, to the Hall bar which will produce a voltage $VQ = k * (J_Q + J_0)$. Applying this one to the qubit No.2 will produce a voltage $2kJ_0 = -2$ (in units of T) at a proper choice of the instrumental parameter k (i.e., the magnitude and the sign of the transport current in a bar J). The voltage is nonzero when qubit is $|0\rangle$, and zero when it is in a state $|1\rangle$. Therefore, when the voltage output from qubit No.1 is applied to the qubit No.2 at the time interval t_1 which is specified in Eq.(11), the qubit No.2 will flip its state or remain idle depending on whether the qubit No.1 is in the $|0\rangle$ or in the $|1\rangle$ states, i.e. the *CNOT* operation.

(c)*Reading the output.* Reading the result, i.e. the population of the qubits when computation concludes can be realized with the same device as one shown in Fig.6 by measuring the voltage on a Hall probe since it is directly proportional to the persistent current, and therefore, by its polarity, distinguishes between the clockwise and the counterclockwise directed currents. The device is analogous to a Stern-Gerlach sensor of Ref.[1], or to the Ramsey-zone measuring devices of the optical beam polarization qubits in Ref.[3].

V. CONCLUSIONS

In conclusion, we presented a new method, using the normal-metal (non-superconducting) persistent-current states [21] in three-site resonantly coupled quantum dots (or in the triple anionic vacancy sites), in perpendicular magnetic and electric fields. The operations of the qu-gate are similar to the previous mechanisms within the qubit sub-

space, but the presence of the auxiliary level manifests the quantum gates without using external switches and contacts. The particular persistent current realization of this three level system however may be hard in practice for many reasons including the necessity of extremely high magnetic field (in the case of a nanoscopic variant with triple vacancies), or very large current-to-voltage conversion ratios for *CNOT* transformation (in case of a mesoscopic variant with triples of quantum dots); the decoherence and the instrumental errors. However, it is very likely that the simplicity of the theoretical mechanism may permit other manifestations to exist. This part is not discussed in the present work in any practical detail except for mentioning that use of high- μ , high- ε materials may resolve some of the problems (see in this respect a recent work [27] in which the Aharonov-Bohm oscillations have been observed in a ring with permalloy). Another trend may be in using of the quantum effects other than the Aharonov-Bohm mechanism (the Berry phase [28], the spin-orbit coupling, etc.) for creating other persistent current structures similar to one studied in present work. The suggested qubit differs from those currently investigated by the quantum-computing community (the photon beams, trapped ions, liquid-state NMR atoms, superconducting Cooper-pair islands or π -Josephson junctions) in the respect of the auxiliary register $|c\rangle$ which provides a possibility of coherent coupling of its operational $|0\rangle$ and $|1\rangle$ registers to (or the mere execution of) the logical reversible *NOT* and *CNOT* gates. Unlike the superconducting (macroscopic) qubits which may be shown to display strong decoherence during switching between the states of opposite direction superconducting Josephson currents (a phase-slip mechanism), the persistent-current qu-bit performs similar transformation (the qu-gate) as a process intrinsically incorporated into their three-level structure and hence not involving the additional decoherence.

The persistent-current scheme is flexible for further modifications including mechanisms of error correction (not discussed in present work), the addition of extra registers for more complex unified bit-gate manipulation, and is expected to better fit the requirements of keeping quantum coherence within the operational cycle of the computer by addressing the memory and the processor registers in the same quantum unit.

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FIGURE CAPTIONS

Fig.1:(a)A sketch of the magnetically focused lines of the magnetic field from the superconducting fluxon trapped in the opening of superconducting foil (S), compressed by ferromagnetic crystal (F) and directed into the interior of PC ring (R); (b)Operational diagram of the qubit at half-flux quantum, $\Phi = \Phi_0/2$. 1,3 - degenerate states carrying opposite currents $\pm j = -c\partial\varepsilon/\partial\Phi$ (full and dashed lines), 2 - zero-current virtual state (a control state, dotted line) which couples the qubit to the logical qu-gate.

Fig.2: A sketch of the bit flip. C is an output coil, V 's are the electrodes creating electric field perpendicular to the magnetic field (normal to sheet) in a qubit.

Fig.3: Energy versus electrostatic potential. 1 and 3 (solid line and dotted line) are the energies which become degenerate at $V_0 = 0$, and 2 (the dashed line) is an energy of the auxiliary control state $|c\rangle$. The arrows indicate the values of the potential V_0 corresponding to the operational points of the bit-flip (i.e. G_1) and G_3 (i.e. Hadamard) gates.

Fig.4: Evolution diagrams of quantum gates G_1 (panel a) and G_3 (panel b). Solid and dashed lines show the time dependence of the population of the states $|0\rangle$ and $|1\rangle$ which are degenerate at $V_0 = 0$. The dotted lines show the time dependence of the auxiliary population. The arrows indicate the “operational point” of the qu-gate, the time of evolution corresponding to the return to the invariant qubit subspace.

Fig.5: Level diagram of the persistent-current qubit. Arrows indicate the virtual transition to the auxiliary state at the fixed-time interval (quenched) Rabi oscillation.

Fig.6: The sketch of a possible physical implementation of the $CNOT$ logical gate. C - current-response loop of a control qubit (qubit No.1), F - flux- $\Phi_0/2$ compensation loop, H

- Hall-effect sensor generating the potential V_0 controlling qubit No.2. J_0 is a fixed current which should be large enough to create a strong voltage output signal ensuring effective control over the second qubit.

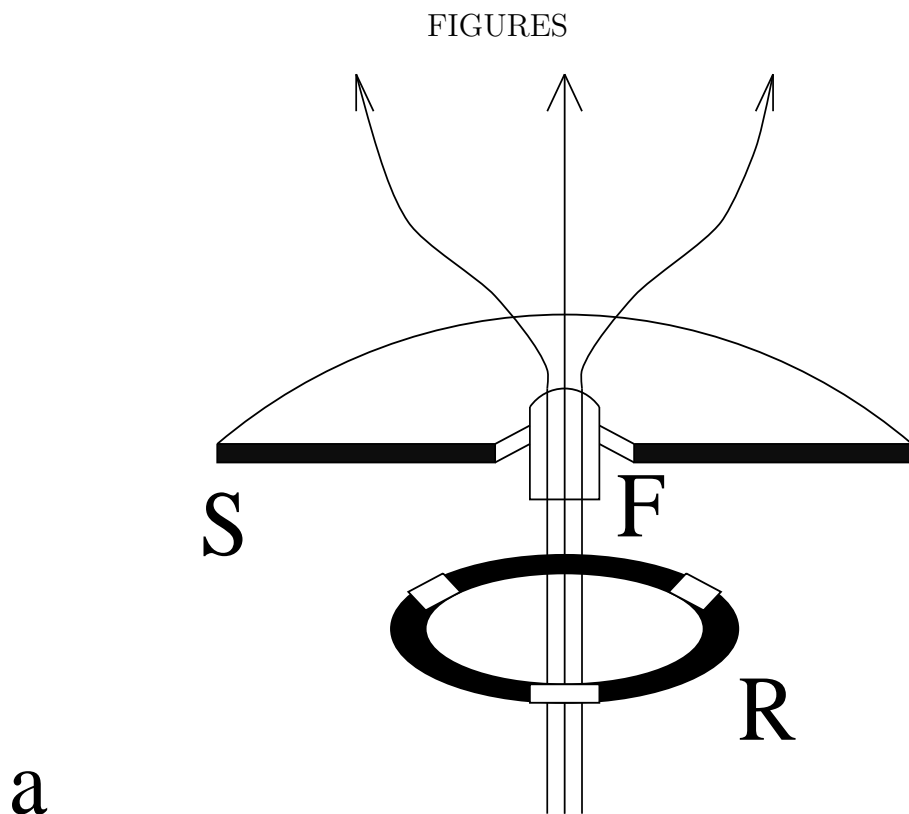


Fig.1a

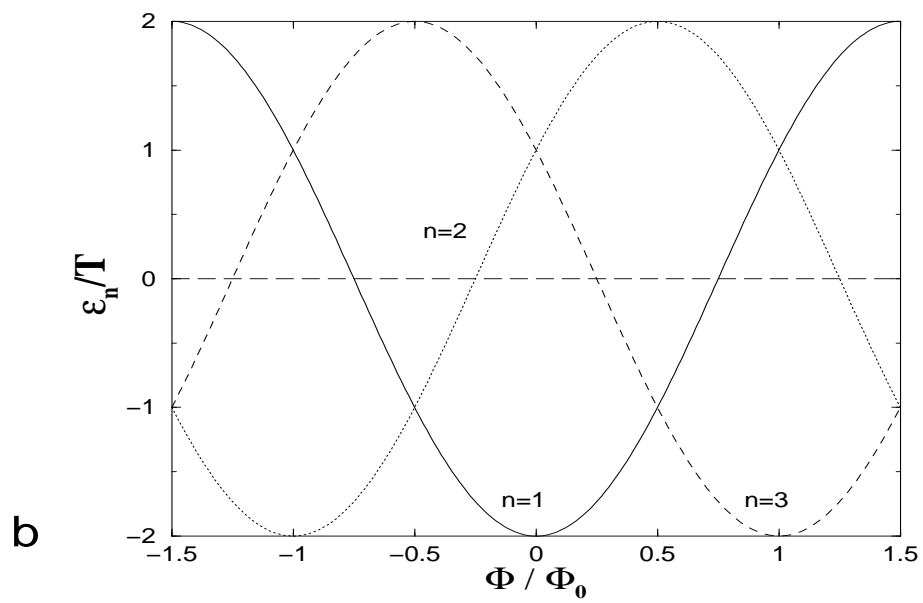


Fig.1b

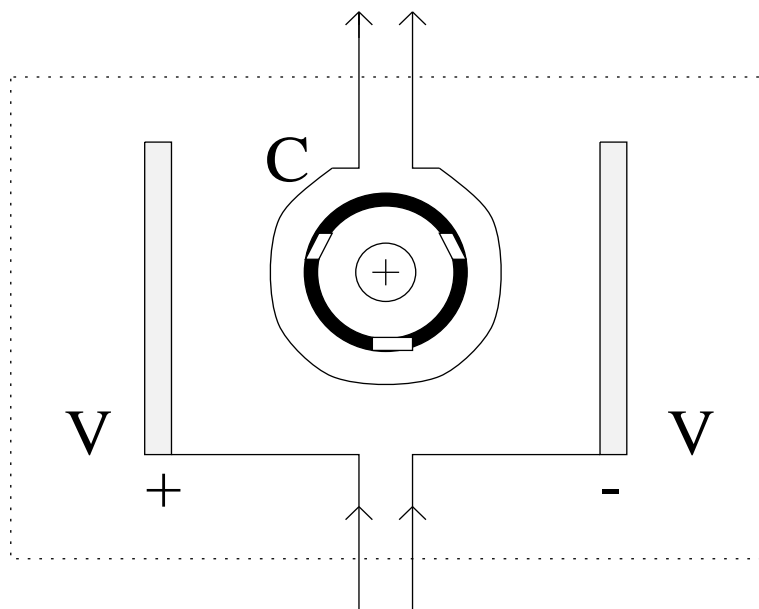


Fig.2

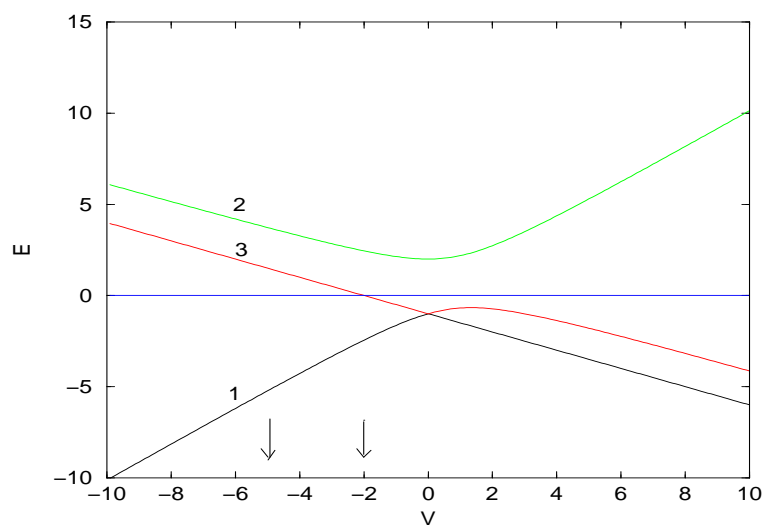


Fig.3

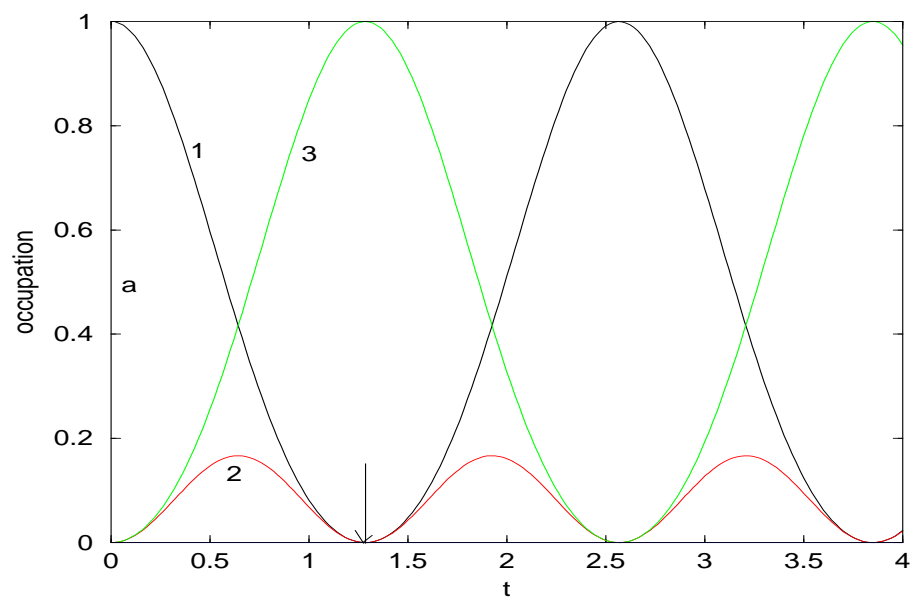


Fig.4a

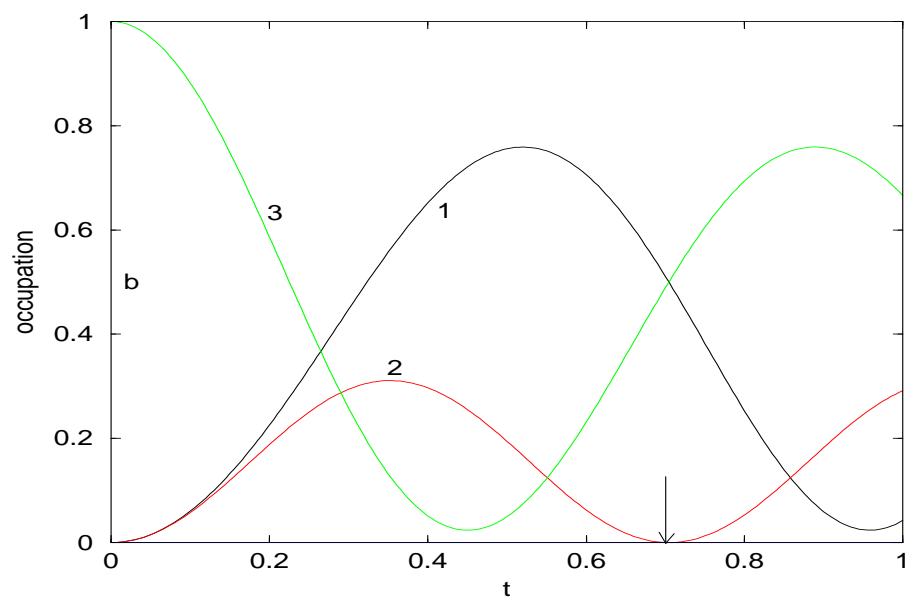


Fig.4b

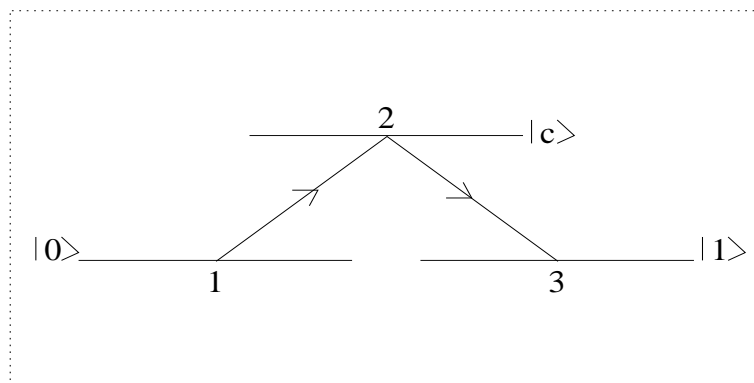


Fig.5

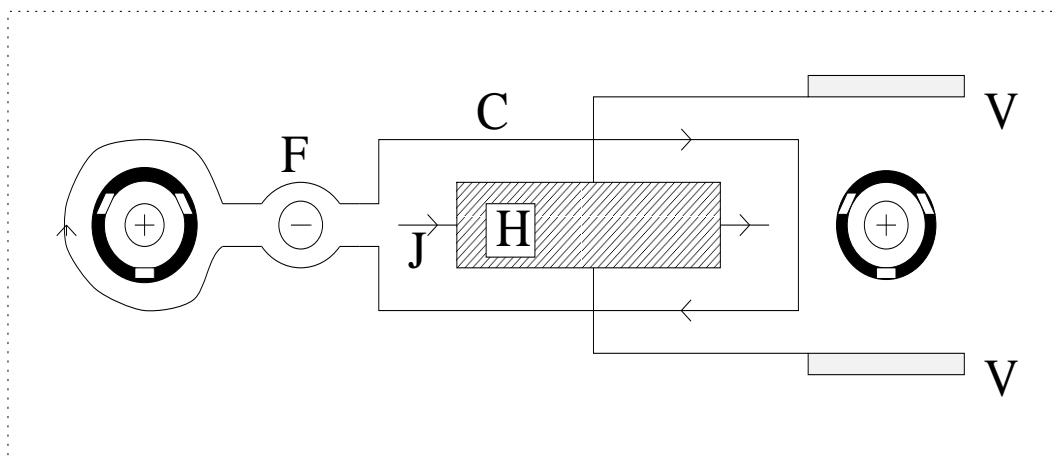


Fig.6